OPTIMISING THE PARALLELISATION OF A HARMONIC BALANCE NAVIER-STOKES SOLVER FOR THE ULTRA-RAPID ANALYSIS OF WIND TURBINE, TURBOMACHINERY AND AIRCRAFT WING PERIODIC FLOWS

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Abstract

COSA is a novel CFD system based on the compressible Navier-Stokes model for unsteady aerodynamics and aeroelasticity of fixed structures, rotary wings and turbomachinery blades. It includes a steady, time domain, and harmonic balance flow solver. All three solvers use a finite volume scheme with structured multi-block grids. All solvers can also use Low-Speed Preconditioning, which enables their use also for low-speed flows, such as those associated with horizontal axis wind turbines. Turbulence effects are incorporated by using the two-equation $K-\omega$ shear stress transport turbulence model of Menter.

COSA has primarily been parallelised using MPI, but there is also a hybrid parallelisation that adds OpenMP functionality to the MPI parallelisation to enable larger numbers of cores to be utilised for a given simulation as the MPI parallelisation is limited to the number of geometric partitions (or blocks) in the simulation. This project worked to optimise these two parallelisation strategies, improving the efficiency of both and therefore reducing the computational time required to compute simulations.

Through a combination of different MPI optimisations we have managed to halve the runtime of the MPI based parallelisation when comparing to the original code for a representative use case. Furthermore, we have optimised the hybrid code so that it is close to 90% efficient when going beyond the number of cores that the MPI can exploit.
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1 Introduction

This report documents the work performed during the dCSE project titled “OPTIMISING THE PARALLELISATION OF A HARMONIC BALANCE NAVIER-STOKES SOLVER FOR THE ULTRA-RAPID ANALYSIS OF WIND TURBINE, TURBOMACHINERY AND AIRCRAFT WING PERIODIC FLOWS”. The project, undertaken at EPCC, The University of Edinburgh, in conjunction with Dr M. Sergio Campobasso from the Systems, Power and Energy Research Division of Glasgow University, aimed to improve the overall performance of the COSA simulation code, thereby reducing the computational resources required to undertake scientific simulations, enabling more efficient use of the resources provided by the HECToR service (and other HPC systems), and reducing the runtime required to undertake simulations for large scale problems.

1.1 COSA

COSA is a novel CFD system based on the compressible Navier-Stokes (NS) model for unsteady aerodynamics and aeroelasticity of fixed structures, rotary wings and turbomachinery blades. The COSA code is written in FORTRAN, and it includes a steady, a time domain (TD) [1-3] and a harmonic balance (HB) [2] flow solver. All three solvers use a finite volume scheme with structured multi-block grids. The integration of the steady and HB equations is based on an explicit multigrid algorithm using a multi-stage Runge-Kutta (RK) smoother. Solution of unsteady problems in the TD is obtained by using Jameson’s dual-time stepping. Turbulent flow effects are modelled by means of the two-equation K-ω Shear Stress Transport (SST) turbulence model of Menter [1].

The COSA suite also uses Low-Speed Preconditioning [2,3], which enables its use also for low-speed flows, such as those associated with horizontal axis wind turbines (HAWT’s). All 3 COSA solvers (steady, TD, and HB) have been parallelised, since their execution requires large amounts of computational resources. There are three different parallelisations of the current code[5,6]:

- OpenMP
- MPI
- Hybrid (mixed OpenMP and MPI)

The OpenMP parallelisation can work for all three solvers, with different parallelisations available over the blocks in the multi-block grids, over the harmonics for the HB solver, and over the grid points for those problems that use low numbers of blocks or harmonics (for instance a single block, TD, simulation).

The MPI parallelisation distributes the blocks of the multi-block grid over the available MPI processes to distribute the work of the simulation. Communication is required between the blocks where data on the edge of blocks (called cuts in COSA) needs to be communicated to neighbouring blocks (halo communications). The maximum number of processes that the MPI parallelisation can use is limited by the number of geometric partitions (grid blocks) in the simulation.
The hybrid parallelisation combines the MPI code with either the harmonic OpenMP parallelisation or the grid point OpenMP parallelisation, depending on the simulation being performed.

1.2 HECToR

HECToR, a Cray XE6 computer, is the UK National Supercomputing Service. This project utilised the Phase 3 incarnation of the system. Phase 3 of HECToR consists of 2816 nodes, each containing two 16-core 2.3 GHz 'Interlagos' AMD Opteron processors per node, giving a total of 32 cores per node, with 1 GB of memory per core. This configuration provides a machine with 90,112 cores in total, 90TB of main memory, and a peak performance of over 800 TFlop/s.

1.3 Test Cases

Three different test cases have been used for understanding the performance and runtime characteristics of COSA, the details of these are provided next.

1.3.1 Test Case 1

This test case is a HB analysis of a heaving and pitching wing designed to extract energy from an oncoming air stream. The 512-block grid has 262,144 cells, and 31 real harmonics are used. This HB analysis has the same memory requirements of a steady flow analysis with more than 8 million cells. Further details on the aerodynamics of this device and the analysis of its efficiency based on COSA time-domain simulations are reported in the articles [1,3].

1.3.2 Test Case 2

The test case is that associated with the HB flow analysis of the blade section at 90% span of a multi-megawatt horizontal axis wind turbine operating in yawed wind. The analysis has been performed using both a coarse and a fine grid, both consisting of 32 grid blocks. The coarse grid has 73728 cells, whereas the fine grid has 654336 cells. Further details on the time-domain and HB COSA analyses of this problem are reported in the article [2].

1.3.3 Test Case 3

Test case 3 is a finer grid version of test case 2. In TC3, the computational grid has 2048 blocks, and the grid has 4,194,304 cells. In the simulations we have used 17 real harmonics.

1.4 Problem to be tackled

Whilst the different parallelisations of COSA enable large simulations to be performed correctly, and in a much shorter time than the serial code, the efficiency of the different parallel implementations is not optimal. The OpenMP parallelisation allows simulations to be run on local computers such as desktop machines, and small shared-memory clusters, however it cannot be used on large scale resources. Furthermore, the OpenMP parallelisation did not scale well even on small numbers of cores. The MPI code is generally efficient, although there are some areas that can be improved, but can only utilise a maximum number of cores equal to the maximum number of grid blocks in the simulation. The hybrid code was developed to address this restriction, enabling parallelisation that use more cores than the number of blocks.
in the simulation, however it was less efficient than the pure MPI parallelisation as it used the inefficient OpenMP functionality as well as the reasonably efficient MPI functionality.

Therefore, this dCSE project aimed to improve COSA in the following two areas to enable it to efficiently be used on HECToR:

1. Optimise MPI communications of the COSA HB solver
2. Optimise the OpenMP and hybrid parallelisation of the COSA HB solver

For the remainder of this report we will outline the current performance of the existing COSA parallelisation in Section 2, discuss the optimisation performed on the MPI code in Section 3, outline the initial optimisation work undertaken on the hybrid code in Section 4, and further advanced hybrid optimisation undertaken in Section 5. We will finish with summary of the work undertaken and the overall performance benefits in the final section, Section 6.

2 Initial Performance

The efficiency of the existing harmonic OpenMP parallelisation has been assessed using test case 1. Figure 1 shows the runtime of the OpenMP parallelisation on a 32-core node of HECToR, and it highlights a poor performance of the OpenMP parallelisation.

![Figure 1: Runtime for the OpenMP code for test case 1](image)

The speedup of the MPI parallelisation of the code on the same test case as the one used for the OpenMP parallel scaling study is shown in Figure 2. The MPI code has much better scaling than the equivalent OpenMP functionality, but it is still not optimal, particularly at larger process counts. The MPI code was designed to provide a one-to-one correspondence to the functionality of the serial code and it sends the dataset of one halo cell at a time rather than the dataset of all halo cells associated with a particular grid cut (boundary between adjacent blocks) at once. Although
important in establishing correctness and confidence in the parallelisation, this simplistic approach is inefficient. In an efficiently implemented MPI code, increasing the size of the grid blocks without altering the overall number of blocks should have a negligible impact on the MPI efficiency, because the overall number of MPI communications does not vary significantly, and the ratio between the time spent for actual computing and communications remains about constant (this is because both the size of the MPI messages and the number of operations of each block increases).

Conversely, in the MPI code, increasing the size of each block reduces the MPI efficiency, because this operation results in a larger number of small MPI messages. However, in the code with an ideal parallelisation there would be a constant number of messages, the size of which depends on the block size, rather than an increase in the number of messages sent. An example is provided by the table below, which reports the results of a scaling test of the existing MPI COSA HB solver on HECToR Phase 2b. Test case 2 was used for these benchmarking runs. As expected, the MPI efficiency is higher for the coarse grid, as a consequence of the substantially larger number of (small) MPI messages present in the fine grid case.

<table>
<thead>
<tr>
<th>Cores</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse grid speed-up</td>
<td>1.00</td>
<td>1.82</td>
<td>3.21</td>
<td>5.56</td>
<td>11.30</td>
<td>21.01</td>
</tr>
<tr>
<td>Fine grid speed-up</td>
<td>1.00</td>
<td>1.80</td>
<td>2.96</td>
<td>4.55</td>
<td>9.34</td>
<td>17.47</td>
</tr>
</tbody>
</table>

Table 1: Scaling of the original MPI code using test case 2 with different mesh sizes

Aside from this issue, the existing MPI HB code has an acceptable performance, also for larger problems, as demonstrated in Error! Reference source not found., where the speed-up of the MPI code is plotted against the number of processes used.

![Figure 2: Parallel speedup of MPI code for test case 1](...
We also looked in detail at the performance of the MPI code, profiling it with the Craypat performance analysis tool available on HECToR. We obtained the following profiling data using 512 MPI processes on test case 1.

<table>
<thead>
<tr>
<th>Samp%</th>
<th>Samp</th>
<th>Imb.</th>
<th>Imb.</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Samp</td>
<td>Samp</td>
<td>Samp%</td>
<td>Function</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>----------</td>
</tr>
<tr>
<td>100.0%</td>
<td>63045.9</td>
<td>--</td>
<td>--</td>
<td>Total</td>
</tr>
<tr>
<td>78.8%</td>
<td>49694.5</td>
<td>--</td>
<td>--</td>
<td>USER</td>
</tr>
<tr>
<td>17.3%</td>
<td>10911.0</td>
<td>596.0</td>
<td>5.2%</td>
<td>blsp_</td>
</tr>
<tr>
<td>14.9%</td>
<td>9415.1</td>
<td>630.9</td>
<td>6.3%</td>
<td>proflux_b_</td>
</tr>
<tr>
<td>6.6%</td>
<td>4143.6</td>
<td>393.4</td>
<td>8.7%</td>
<td>vflux_b_</td>
</tr>
<tr>
<td>4.7%</td>
<td>2933.7</td>
<td>739.3</td>
<td>20.1%</td>
<td>src_bhb_</td>
</tr>
<tr>
<td>4.1%</td>
<td>2554.6</td>
<td>289.4</td>
<td>10.2%</td>
<td>tridi_</td>
</tr>
<tr>
<td>2.7%</td>
<td>1709.7</td>
<td>368.3</td>
<td>17.7%</td>
<td>muscl_bi_</td>
</tr>
<tr>
<td>2.7%</td>
<td>1698.2</td>
<td>183.8</td>
<td>9.8%</td>
<td>cirs_b_</td>
</tr>
<tr>
<td>2.7%</td>
<td>1686.2</td>
<td>155.8</td>
<td>8.5%</td>
<td>bresid_</td>
</tr>
<tr>
<td>2.6%</td>
<td>1660.3</td>
<td>213.7</td>
<td>11.4%</td>
<td>muscl_bj_</td>
</tr>
<tr>
<td>1.6%</td>
<td>1020.4</td>
<td>128.6</td>
<td>11.2%</td>
<td>eig_ns_p_</td>
</tr>
<tr>
<td>1.6%</td>
<td>1011.3</td>
<td>190.7</td>
<td>15.9%</td>
<td>prec_bres_</td>
</tr>
<tr>
<td>1.5%</td>
<td>936.2</td>
<td>198.8</td>
<td>17.5%</td>
<td>lm_bviscoff_</td>
</tr>
<tr>
<td>1.5%</td>
<td>922.2</td>
<td>179.8</td>
<td>16.3%</td>
<td>bq_der_i_</td>
</tr>
<tr>
<td>1.2%</td>
<td>785.1</td>
<td>103.9</td>
<td>11.7%</td>
<td>bdeltat_</td>
</tr>
<tr>
<td>1.2%</td>
<td>782.9</td>
<td>210.1</td>
<td>21.2%</td>
<td>bq_der_j_</td>
</tr>
<tr>
<td>1.2%</td>
<td>727.7</td>
<td>101.3</td>
<td>12.2%</td>
<td>limit_j_</td>
</tr>
<tr>
<td>1.1%</td>
<td>708.3</td>
<td>212.7</td>
<td>23.1%</td>
<td>limit_i_</td>
</tr>
<tr>
<td>1.1%</td>
<td>698.0</td>
<td>124.0</td>
<td>15.1%</td>
<td>b_rtst_</td>
</tr>
<tr>
<td>1.0%</td>
<td>633.1</td>
<td>107.9</td>
<td>14.6%</td>
<td>cut_q_</td>
</tr>
<tr>
<td>14.7%</td>
<td>9292.6</td>
<td>--</td>
<td>--</td>
<td>MPI</td>
</tr>
<tr>
<td>10.2%</td>
<td>6412.1</td>
<td>3715.9</td>
<td>36.7%</td>
<td>MPI_WAITANY</td>
</tr>
<tr>
<td>2.7%</td>
<td>1704.2</td>
<td>170.8</td>
<td>9.1%</td>
<td>MPI_ISEND</td>
</tr>
<tr>
<td>6.4%</td>
<td>4058.8</td>
<td>--</td>
<td>--</td>
<td>ETC</td>
</tr>
<tr>
<td>1.4%</td>
<td>886.5</td>
<td>127.5</td>
<td>12.6%</td>
<td>__c_mzero8</td>
</tr>
<tr>
<td>1.2%</td>
<td>731.8</td>
<td>251.2</td>
<td>25.6%</td>
<td>__int_malloc</td>
</tr>
</tbody>
</table>

We can see that the majority of the runtime of COSA (around 80%) for this testcase is spent in user routines (i.e. the subroutines in the COSA project), with around 15% of the runtime in MPI communications. Furthermore, we can see that there are 10 subroutines that take up around 60% of the overall runtime of the code. Therefore, we can identify those routines that are critical for optimising the OpenMP performance (and therefore ensuring that the hybrid code has good performance), and also which parts of the communication functionality are dominating the MPI performance enabling us to effectively target our work on the MPI optimisation. It
should be noted that the above profiling data was collected with the I/O in COSA
turned off, so I/O costs have not been profiled.

3 Optimised MPI Parallelisation

The primary focus for this work was to optimise the inefficient MPI communications
in COSA. The existing code utilises non-blocking MPI communications, but for a
large simulation there can be as many as 5,000 messages sent between
communicating processes at each Runge-Kutta step. Optimising this was one of the
first steps in the dCSE. However, there were a number of other areas that were also
identified for optimisation in this work. A number of places in the COSA code use
hand coded linear algebra functionality, it was decided to replace this with the same
functionality implemented in a linear algebra library. Furthermore, for large
simulations the I/O was seen to take up a large portion of the runtime so we also
looked at optimising that. The following subsections will go into these optimisations
in more detail.

3.1 Rationalise MPI Communications

As mentioned in the introduction to this section the current MPI parallelisation can
require a large number of messages to be send and received to communicate “halo” or
cut data to neighbouring processes. This is because the existing code sends small
parts of the boundary data to neighbouring processes at a time, with an example of
this shown in the following pseudo code:

```plaintext
do i = 0, boundary length
  if(myblock1 .and. myblock2) then
    do n = 0, 2*nharms
      do ipde = 1, npde
        copy 1st part of q2 to q1
        copy 2nd part of q2 to q1
      end do
    end do
  else if(myblock1) then
    receive 1st part of q1 from remote process
    receive 2nd part of q1 from remote process
  else if(myblock2) then
    send 1st part of q2 to remote process
    send 2nd part of q2 to remote process
  end if
end do
```

Note that in the above pseudo code we can see that the MPI communications have
already been partially optimised, as they don’t send a message for each element of the
n and ipde loops, they aggregate the data to be sent or received into an array and
then send that array, as shown in the following code (which implements one of the
send steps in the pseudo code above):

```plaintext
datasize = npde*((2*nharms)+1)
tempindex = 1
do n = 0, 2*nharms
  do ipde = 1, npde
```


sendarray(tempindex,localsendnum) =
&                q2(in1,jn1,ipde,n)
tempindex = tempindex + 1
end do
end do

call sendblockdata(sendarray(1,localsendnum),iblk1,
&                iblk2,datasize,sendrequests(localsendnum))

However, as the send and receive functionality is within a loop, and for that loop the send and receive processes do not change (the same sender and receiver are involved in all the communications for a given invocation of the loop) it is possible to reduce all these send and receives down to one send and one receive by further aggregating the data into a single send array and using a single receive array.

We implemented such a scheme, which aggregated together communications, at the expense of extra data storage (required to store the aggregated data), and tested it with test case 1. All the communication functionality is contained in routines named cutman_in COSA. The results for HECToR are shown in Figure 3.

![HECToR Runtimes](https://example.com/figure3.png)

**Figure 3: Runtime for the new MPI code vs the old MPI code on HECToR using test case 1**

We can see from Figure 3 that the optimised MPI communication strategy has not improved the performance of the code; indeed the original MPI code is slightly faster at 256 and 512 cores whereas the new MPI code is slight faster at lower core counts. This is contrary to what we expected so we investigate what was happening in the code in more detail using the Scalasca[4] profiling tool.

We ran performance profiling using the test data outlined in the previous Figures and discovered that the new MPI code does indeed dramatically reduce the number of MPI messages sent by the cutman_routines. Figure 4 and 5 show examples of the output of the Scalasca tool showing message counts and data transfer sizes for the old and new codes.
Figure 4: Communications profile, old MPI code using test case 1
We can see from these Figures that the total amount of data sent using both versions of the code is exactly the same (see the left hand column, under the “Bytes transferred” heading). However, if we look at the “Communications” metric which outlines how many messages are sent, we can see that the new code sends two orders of magnitude fewer messages than the old code ($1.39 \times 10^9$ vs $2.47 \times 10^7$ for the sends and receives).
Figure 6: Timings profile, old MPI code using test case 1
Figures 6 and 7 present profile information from the same runs as Figures 4 and 5, however Figures 6 and 7 now focus on the runtime information (how much time is spent in routines) rather than how often routines are called. From these we can see that the new code does reduce the overall time spent in the messages sent for the halo exchange (the point-to-point message times are $6.19 \times 10^4$ vs $5.45 \times 10^4$) however, compared to the overall runtime (approximately $5.2 \times 10^5$) this is not a large saving. Furthermore, the saving in the point-to-point time is balanced by an increase in the time for the collective communications (from around $1 \times 10^4$ for the old code to $1.57 \times 10^4$ for the new code).
We can see that optimising the point-to-point communications when there are collective communications occurring every timestep is simply moving the place where the synchronisation (and therefore loss of performance) occurs from the halo communication exchange to the collective (global) communications. Furthermore, the new MPI code uses a different approach to the original code. In the original code the halo data associated with each cut owned by a process is sent as the cut is computed. This means that large numbers of messages are sent, however it does have the benefit that the communications associated with sending the cut data are spread out through the cut calculations/computations. The new code collects all the data associated with all the cuts a process has to communicate and then sends them all at once after all the cuts have been processed. This concentrates the communication into a much smaller section of the code and may cause contention for the communication network by (as a consequence of the approach taken) meaning processes tend to all communicate at the same time rather than the communications being spread out.

However, the performance data we collected was contrary to what we expected and had experienced on previous versions of HECToR (when COSA was run on phase2b of HECToR the MPI communications were seen to be more dominant), therefore we ran the same scaling tests as outlined in Figure 3 on a different machine (a large Bull supercomputer with 2 x 8 core Intel Xeon processors per node and an Infiniband QDR interconnect) and collected the performance data shown in Figure 8.

![Figure 8: Runtime for the new MPI code vs the old MPI code on Bull supercomputer using test case 1](image)

We can see from Figure 8 that the performance profile is significantly different to that of HECToR. Here, combining the MPI messages has significantly improved the overall performance at all core counts (it should be noted that we could not complete the 16 core original MPI job in the time limit for jobs on this machine so it is not included in the Original MPI line), with the optimised code runtime being around two orders of magnitude quicker than the original code.

We can, therefore, conclude that the network on HECToR is very good at performing lots of small messages, so much so that the aggregation we performed did not
significantly improve performance on HECToR. However, for other systems, and even potentially for ARCHER (depending on what hardware is chosen for that machine), HECToR’s replacement, this is a very beneficial change.

### 3.2 Collective Optimisations

Following on from the MPI optimisation we documented in the previous section we looked at the collective communications used within COSA. The main routine that uses MPI collective communication functions in COSA is `combineforces`, the body of which is outlined below:

```plaintext
do i=1,nbody
    temparray(1) = cl(n,i)
    temparray(2) = cd(n,i)
    datalength = 2
    if(functag.eq.3) then
        temparray(3) = cm(n,i)
        datalength = 3
    end if
    call realsumallreduce(temparray,datalength)
    cl(n,i) = temparray(1)
    cd(n,i) = temparray(2)
    if(functag.eq.3) then
        cm(n,i) = temparray(3)
    end if
end do
```

Studying this code it is evident that the code is potentially undertaking more all reduce operations than is necessary, especially if `nbody` is large. Furthermore `combineforces` is called from within a loop that iterates over the number of harmonics (this sets the `n` variable in the code above), meaning that the all reduce is called `nbody*((2*nharms)+1)` times.

By moving where `combineforces` is called from and aggregating the data to be communicated in the all reduce operations it is straightforward to combine these all reduce calls into a single all reduce for each call of the `combineforces` routine, as shown below:

```plaintext
j = 1
do k = 0,2*nharms
    do i=1,nbody
        temparray(j) = cl(k,i)
        j = j +1
        temparray(j) = cd(k,i)
        j = j +1
        temparray(j) = cm(k,i)
        j = j +1
    end do
end do
call realsumallreduce(temparray,j-1)
```
j = 1
do k = 0,2*nharms
  do i=1,nbody
    cl(k,i) = temparray(j)
    j = j +1
    cd(k,i) = temparray(j)
    j = j +1
    cm(k,i) = temparray(j)
    j = j +1
  end do
end do

As with the previous communication aggregation we have performed this at the expense of extra memory requirements for the combineforces routine, however these are not significant so do not adversely impact the overall memory footprint of the code, even for high nbody and harmonic sizes.

We profiled the new code using scalasca as before, and the results are shown in Figures 9 and 10
Figure 9: Collective communication profile for the original MPI code using test case 1
We can see from comparing the number of collective communications performed in Figures 9 and 10 that the number has reduced by an order of magnitude, from $1.37 \times 10^6$ in Figure 9 to $1.73 \times 10^5$ in Figure 10, whilst the amount of data transferred (shown under the bytes transferred section) has stayed the same.

### 3.3 Rationalise MPI I/O

All the profiling, performance analysis, and optimisation work we have outlined in the previous sections has been focussing on the main computational and communication parts of COSA. Particularly, we have been using COSA with the data output turned off. However, for any real user of the code this is not an option, the data that is
written into the output files records the simulation undertaken by the code and enables analysis and understanding of those simulations.

COSA produces a number of different output files, but for optimisation there are two types of file that are important, as they are the largest and require the most time to write; the flowtec files and the restart file. COSA produces a single restart file at the end of the simulation (or more frequently if requested by the user) which can be used to restart the simulation from the point the restart file was written. It also produces one flowtec file per real harmonic at the end of the simulation. The flowtec files contain the solution in a format suitable for use with the commercial CFD postprocessor and flow visualisation software TECOLOT.

When large simulations are executed the output can be extremely large, with the restart file being many gigabytes (GB) in size and each flowtec file being close to a GB in size. We have noticed when undertaking large simulation with the I/O functioning that writing these files was taking a significant fraction of the total runtime.

Therefore, we worked on optimising the I/O used for writing the main output files to ensure that the I/O is being performed efficiently. The existing code does use parallel I/O functionality, calling MPI I/O routines to perform the output from all processes at once. However, the I/O is performed, as shown in the example below, through individual writes of data elements to the file one at a time:

```
call setupfile(fid(n),disp,MPI_INTEGER) 
call mpi_file_write(fid(n), 4*doublesize,1, 
   MPI_INTEGER,MPI_STATUS_IGNORE,ierr) 
disp = disp + integersize 
call setupfile(fid(n),disp,MPI_DOUBLE_PRECISION) 
call mpi_file_write(fid(n),x(i,j,n),1, 
   MPI_DOUBLE_PRECISION,MPI_STATUS_IGNORE, ierr) 
disp = disp + doublesize 
call setupfile(fid(n),disp,MPI_DOUBLE_PRECISION) 
call mpi_file_write(fid(n),y(i,j,n),1, 
   MPI_DOUBLE_PRECISION, MPI_STATUS_IGNORE, ierr) 
disp = disp + doublesize 
call setupfile(fid(n),disp,MPI_DOUBLE_PRECISION) 
call mpi_file_write(fid(n),rho,1, 
   MPI_DOUBLE_PRECISION, MPI_STATUS_IGNORE, ierr) 
disp = disp + doublesize 
call setupfile(fid(n),disp,MPI_DOUBLE_PRECISION) 
call mpi_file_write(fid(n),ux,1, 
   MPI_DOUBLE_PRECISION, MPI_STATUS_IGNORE, ierr) 
disp = disp + doublesize 
call setupfile(fid(n),disp,MPI_INTEGER) 
call mpi_file_write(fid(n), 4*doublesize,1, 
   MPI_INTEGER,MPI_STATUS_IGNORE,ierr) 
```

Where the setupfile subroutine invokes the MPI_FILE_SEEK function.
This use of MPI I/O is not optimal; generally MPI I/O gives the best performance when large amounts of data are written in a single call to the file. However, the way the data is structured in COSA, and the format of the output files, prohibits doing this.

It is important to the developers and users of COSA that the output files of the serial and parallel version of the code are the same so in the scope of this project we did not have the effort to change the way it currently writes the data. However, we can optimise the current functionality, aggregating the data to be written into arrays and then writing that data all at once. An example of this optimisation of the I/O code outlined above is provided below:

```fortran
  call setupfile(fid(n),disp,MPI_INTEGER)
  call mpi_file_write(fid(n), 4*doublesize,1,
        &   MPI_INTEGER,MPI_STATUS_IGNORE,ierr)
  disp = disp + integersize
  tempdata(tempindex) = x(i,j,n)
  tempindex = tempindex + 1
  tempdata(tempindex) = y(i,j,n)
  tempindex = tempindex + 1
  tempdata(tempindex) = rho
  tempindex = tempindex + 1
  tempdata(tempindex) = ux
  tempindex = tempindex + 1
  call setupfile(fid(n),disp,MPI_DOUBLE_PRECISION)
  call mpi_file_write(fid(n),tempdata(1),tempindex-1,
        &   MPI_DOUBLE_PRECISION, MPI_STATUS_IGNORE, ierr)
  disp = disp + doublesize*(tempindex-1)
  call setupfile(fid(n),disp,MPI_INTEGER)
  call mpi_file_write(fid(n), 4*(tempindex-1),1,
        &   MPI_INTEGER,MPI_STATUS_IGNORE,ierr)
```

Testing the new I/O functionality using test case 1 we collected the following data:

<table>
<thead>
<tr>
<th></th>
<th>Runtime (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original MPI Code (No I/O)</td>
<td>421</td>
</tr>
<tr>
<td>Original MPI Code</td>
<td>701</td>
</tr>
<tr>
<td>New MPI Code</td>
<td>547</td>
</tr>
</tbody>
</table>

Table 2: I/O functionality runtime using test case 1

We can see that for the simulation performed (using 512 processes and running for 250 iterations of the simulation) that the original I/O functionality imposed a significant overhead on the overall simulation time, increasing the runtime by around 60%. The optimised I/O functionality reduces this overhead to around 30%, more than halving the time required to output data to disk. For this simulation the amount of data outputted is not very large, the restart file is around 350 MB and there are 31 flowtec files of 20 MB each (so overall just less than 1 GB is written to disk).

However, if we consider a larger simulation then the savings are even more significant. The previous simulation (test case 1) had a 512-block grid with 262,144
cells, and 31 real harmonics, but the simulations currently being studied using COSA typically have a million or more cells. We benchmarked the new MPI functionality using test case 3 (which produces a restart file of around 2.6 GB and 15 flowtec files each 281 MB in size), only undertaking a single iteration of the simulation. The resultant runtimes on 512 and 2048 cores are shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Original I/O Code (seconds)</th>
<th>New I/O Code (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>512 processes</td>
<td>1444</td>
<td>904</td>
</tr>
<tr>
<td>2048 processes</td>
<td>1147</td>
<td>801</td>
</tr>
</tbody>
</table>

Table 3: Performance of the new I/O functionality using test case 3

We also discovered a bug in the parallel I/O code when undertaking this work. When writing very large restart files (above ~4 GB) the original code did not work. It was ascertained that the code that calculated the locations in the file for each MPI process to write data to using a FORTRAN integer datatype. However, on HECToR (and many other machines) this defaults to a 4-byte integer datatype, and therefore cannot be used to calculate address bigger than $2^{32}$ bytes of memory (around 4 GB). We fixed this by specifying these offset addresses as 8-byte integers in the code, enabling COSA to deal with larger simulations properly.

We evaluated the overall performance improvement from all the MPI optimisations we have detailed in this section comparing the performance of the original code with the latest MPI code, and plotting the results in Figure 11 below.

![Figure 11: Runtime of the original and optimised MPI codes using test case 1](image)

We have significantly improved the performance of the code at larger core counts, with the runtime of the optimised MPI code at 256 and 512 cores around half the time of the original MPI code.
3.4 Linear Algebra Routines

During the performance evaluation of COSA for this project it was recognised that there were a number of subroutines in the code implementing standard linear algebra functionality, particularly:

- Matrix inversion
- Matrix-vector products

The harmonic balance Navier-Stokes equations can be viewed as a set of 
\((2*nharms)+1\) steady problems, all coupled by a source terms depending on the same number of steady flow solutions, which represent \((2*nharms)+1\) equally-spaced snapshots of the sought nonlinear periodic flow [2]. The multigrid HB solver of COSA can be operated in two modes, using either a fully explicit Runge-Kutta (FERK) smoother, or a Point-Implicit Runge-Kutta (PIRK) smoother. In the PIRK integration, the HB source term is treated implicitly at each Runge-Kutta stage. This results in the entire solution update process requiring the solution of \(ncell\) linear systems, each of dimension \(npde*((2*nharms)+1)\) at every Runge-Kutta stage. However, the use of the PIRK HB integration allows the use of substantially larger pseudo-time steps for time-marching the solution with respect to the conventional FERK HB integration, without any loss of numerical stability. As a consequence, a given convergence level of the HB solution can be achieved using a substantially lower number of MG iterations using the PIRK rather than the FERK integration.

The original COSA code solved these \(ncell\) systems using hand-written linear algebra routines. These were replaced by the corresponding routines in the LAPACK and BLAS libraries to ensure that optimal code for these functions were being used. All test cases presented in this report used relatively small time-steps and therefore were run using the FERK HB integration. However, the PIRK functionality is now routinely used in production runs of the COSA HB solver, since it significantly reduces runtimes due to the use of fewer MG cycles. In order to demonstrate the computational performance improvement achieved by using the LAPACK routines, test case 1 has been run on HECToR also using the PIRK HB integration. It has emerged that, at 512 processes, the new version of the code (using LAPACK routines) runs in around half the time of the original code (~740 seconds vs ~1400 seconds)

The FERK or PIRK integration are selecting by choosing the appropriate flag \((rkex\) or \(rkim\)) in the input file of COSA.

4 Optimised Hybrid Parallelisation

Using the information obtained in the profiling of the original code we worked to optimise the OpenMP functionality in COSA. The profiling data shown in Section 2 highlighted the fact that only a small number of routines are heavily used for harmonic balance simulations.

The original hybrid code, which used the original OpenMP parallelisation, could be built to parallelise over blocks, harmonics, or grid cells, depending on the type of simulation being performed. However, as the key strength of COSA is its harmonic
balance functionality we decided to focus the hybrid optimisation on harmonic balance simulations.

Removing the original OpenMP functionality and re-implementing OpenMP simply for the key routines we have identified enabled us to optimise the code by removing OpenMP parallel regions from parts of the code that are used in the harmonic balance simulations but do not consume much of the runtime (and therefore do not have enough work within them to justify the overheads imposed when using OpenMP parallelisations).

The following table contains all the routines in COSA that had OpenMP added during this work. In the table the subroutine column lists those routines that have OpenMP parallel regions in them. The related subroutines column lists those routines that have orphaned OpenMP do directives in them which are called by the routines lists in the subroutine column.

<table>
<thead>
<tr>
<th>File</th>
<th>Subroutine</th>
<th>Related Subroutines</th>
</tr>
</thead>
<tbody>
<tr>
<td>cosa.f</td>
<td>resid</td>
<td>bresid</td>
</tr>
<tr>
<td>cosa.f</td>
<td>deltat</td>
<td>bdeltat</td>
</tr>
<tr>
<td>cosa.f</td>
<td>cirshet</td>
<td>cirshet</td>
</tr>
<tr>
<td>cosa.f</td>
<td>precres</td>
<td>prec_bres</td>
</tr>
<tr>
<td>cosa.f</td>
<td>src_hb</td>
<td>src_bhb</td>
</tr>
<tr>
<td>flux.f</td>
<td>roflux</td>
<td>roflux_b</td>
</tr>
<tr>
<td>flux.f</td>
<td>proflux</td>
<td>proflux_b</td>
</tr>
<tr>
<td>flux.f</td>
<td>vflux</td>
<td>vflux_b</td>
</tr>
<tr>
<td>flux.f</td>
<td>muscl</td>
<td>muscl_bi</td>
</tr>
<tr>
<td></td>
<td></td>
<td>limit_i</td>
</tr>
<tr>
<td></td>
<td></td>
<td>muscl_bj</td>
</tr>
<tr>
<td></td>
<td></td>
<td>limit_j</td>
</tr>
<tr>
<td>flux.f</td>
<td>q_der</td>
<td>bq_der_i</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bq_der_j</td>
</tr>
<tr>
<td>flux.f</td>
<td>rtst</td>
<td>b_rtst</td>
</tr>
<tr>
<td>losp.f</td>
<td>lsp</td>
<td>blsp</td>
</tr>
<tr>
<td>losp.f</td>
<td>lm_cutoff_pg</td>
<td>lm_bcutoff_pg</td>
</tr>
<tr>
<td>losp.f</td>
<td>lm_viscoff</td>
<td>lm_bviscoff</td>
</tr>
</tbody>
</table>

However, we also recognised that the way the OpenMP code has been added to COSA also was not efficient. In general the loops in the code that compute over the harmonics of the simulation are called within subroutines which are called for each block in the simulation, and in each subroutine there can be a number of separate loops over harmonics. Simply parallelising each harmonic loop with an OpenMP parallel do directive meant that there were a lot of places that OpenMP parallel regions were started and finished in the code. There is an overhead associated with starting and finishing a parallel region in OpenMP, therefore we re-engineered the OpenMP code we had added to reduce these overheads.

This reduction in overheads was achieved by hoisting (moving to a higher level in the program) the parallel regions from within the subroutines where the harmonic loops are, to the calling subroutine, as shown in the two code samples in Appendix A.
We also implemented “first touch” initialisation functionality to ensure that data is initialised on the cores that will be processing it. The original code zeroed all the data arrays when they were allocated, and the allocation is done on the master thread. We have altered the zeroing of the arrays so it is done in parallel, following the parallelisation pattern that is used in the rest of the code.

Comparing the performance of the new OpenMP code with that of the original harmonic OpenMP parallelisation we obtained the scaling data shown in Figure 12. It should be noted that this data was collected with I/O turned on, using test case 1, which is not an ideal test case for the OpenMP harmonic parallelisation.

![Figure 12: Runtime of the old and new OpenMP harmonic parallelisation using test case 1](image)

It should also be noted that due to the hardware used in HECToR, which is AMD Interlagos processors, it is difficult to compare the performance of 1 process using a single core on a processor to 2 processes using 2 cores on a processor as the processor is made up of 8 modules, each of which can run 2 processes (giving the 16 cores in a processor, and HECToR has 2 processor per node giving 32 cores per node), but they share the floating point unit and memory caches and bandwidth. The cores can also change their computing frequency if not all the cores on the processor are being used. Therefore, the performance of the program using a single core on a processor (whilst the rest are idle) may be significantly higher than if 2 or more of the cores are being used as using a single core only potentially gives that process access to more floating point hardware and memory bandwidth than is available when 2 processes are running on a module. This means that calculating the scaling of a parallel code based on the performance of the code on a single core may not be ideal.

Using this optimised OpenMP implementation improves the performance of the hybrid code, as shown in the table next, which provides a comparison of the performance of the hybrid and MPI codes for the old OpenMP functionality and the optimised functionality (using test case 1).

<table>
<thead>
<tr>
<th>MPI Runtime</th>
<th>Old Hybrid Runtime</th>
<th>New Hybrid Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>1693</td>
<td>2670.13</td>
<td>2215.69</td>
</tr>
</tbody>
</table>

Table 4: Performance of the hybrid code using test case 1 on 512 cores
The data in the above table is collected using a simulation with 512 blocks so the MPI code is using 512 MPI tasks and the hybrid code is using 128 MPI tasks each with 4 threads. This means that each MPI task in the hybrid case has 4 blocks of the simulation, which is not ideal for the performance of the new hybrid code. This hybrid code is designed to work best when the number of blocks per MPI task is 1, so we also ran the hybrid code using 512 MPI tasks each with 4 OpenMP threads (using a total of 2048 cores), the results of which are show in the following table (using test case 1).

<table>
<thead>
<tr>
<th></th>
<th>MPI (512 cores)</th>
<th>Old Hybrid (2048 cores)</th>
<th>New Hybrid (2048 cores)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runtime (seconds)</td>
<td>1693</td>
<td>772.16</td>
<td>520</td>
</tr>
<tr>
<td>Scaling (compared to the MPI code)</td>
<td>N/A</td>
<td>2.19</td>
<td>3.26</td>
</tr>
</tbody>
</table>

Table 5: Performance of the hybrid code using test case 1 on 2048 cores

We can see that we have improved the performance of the hybrid code compared to the original hybrid code. However, there is still a performance gap between the ideal scaling of the hybrid code and the currently scaling of the hybrid code. Ideally, when using 4 times the number of cores (as is the case in the example above) the scaling would be 4 times the pure MPI code, whereas the new hybrid code has a scaling of 3.26.

5 Advanced Hybrid Functionality

For the hybrid code we have parallelised the key computational routines using OpenMP, but as outlined in Section 4 the performance is still not ideal for the hybrid code. However, there are two areas in the code that haven’t been parallelised with OpenMP:

- I/O
- MPI Communication

The MPI communications are performed over a loop of the cut, or halo, data. Each cut is independent so they can be performed by separate threads. However, as they involve MPI communications then we need to ensure that we are using the threaded version of the MPI library using the function `MPI_INIT_THREAD` rather than the usual `MPI_INIT` function. Furthermore, we need to ensure that the MPI library being used can support `MPI_THREAD_MULTIPLE` (individual OpenMP threads can perform MPI communications).

The I/O undertaken through the MPI code used MPI I/O functionality. In general the I/O operations are independent for each block and then each harmonic within the block. However, there are a number of collective operations (operations that all processes must be involved in) in the I/O functionality, particularly opening and closing files. To enable the OpenMP threads to be able to write to the restart and flowtec files independently we needed to ensure that all the threads are involved in the opening of the files so they each have a separate file handle to write. Therefore, we implemented a hybrid file opening and closing strategy as follows:

```c
!$OMP DO ORDERED
    do i=1,omp_get_num_threads()
```
!$OMP ORDERED
call openfile(fid,'restart',iomode)
!$OMP END ORDERED
end do
!$OMP END DO

Where openfile calls MPI_FILE_OPEN which is a collective operations and therefore has to be called from all the OpenMP threads in the order sequence shown above. The only other modification that needs to be made to enable file writing from the OpenMP threads was to ensure that they could correctly calculate where each harmonic needs to be written to (rather than each block as was the case previously).

With these optimisations in place the code was once again benchmarked and the following results collected (using test case 1):

<table>
<thead>
<tr>
<th></th>
<th>MPI (512 cores)</th>
<th>Original Hybrid (2048 cores)</th>
<th>New Hybrid (2048 cores)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runtime (seconds)</td>
<td>1693</td>
<td>772.16</td>
<td>460</td>
</tr>
<tr>
<td>Scaling (compared to the MPI code)</td>
<td>N/A</td>
<td>2.19</td>
<td>3.68</td>
</tr>
</tbody>
</table>

Table 6: Optimised hybrid runtime using test case 1

We can see that the hybrid code performance has improved, with close to ideal scaling being obtained for this benchmark case, even when full functionality (i.e. including I/O) is used. However, it should be noted that to achieve this performance it is necessary to change the lustre stripe count (which specifies how many I/O servers can be used to write the file) for the restart file. By default the restart file has is setup to use a single I/O server (a stripe count of 1), and this gives ideal performance of the MPI code and the old hybrid code. However, when the restart I/O functionality is parallelised with OpenMP the default stripe count does not give ideal performance, so we changed this stripe count to -1 which enables all available I/O servers to be used. This gave better performance than the original stripe count.

The parallelisation of the writing of the flowtec files over harmonics with OpenMP does not require this as there are multiple flowtec files which can be distributed to different I/O servers by the lustre file system.

6 Summary

We have undertaken a range of optimisation work on the COSA simulation code. The MPI optimisation work we performed has resulted in the new code scaling much better than the original MPI code, with the overall runtime being reduced to half that of the original code for a representative test case at 256 and 512 cores on HECToR. This represents a significant saving in computational resources.

We have also worked to optimise the hybrid version of the code. The original hybrid code gave around a halving in computational runtime for four times the computational resources, with an operating efficiency of around 55%. The new hybrid code operates at around 90% efficiency for the same test case on HECToR, again representing a significant saving in computational resources, and furthermore enabling users to complete simulations in around half the time that the original code took for the same problem.
Another outcome of this project is that COSA has now been installed as a package, available to all users, on HECToR. MPI and hybrid executables are available, along with a user guide describing the code and how to use it, and example simulations to enable interested parties to experiment with the software. The executables include all the optimisations undertaken in this project.

All the code modifications performed in this project have been incorporated back into the main COSA source code, and extensively tested to ensure they have not altered the correctness of the COSA simulations.

7 Acknowledgements
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References


Appendix A
Below is an example of the original OpenMP harmonic parallelisation, including the routine that calls the routine that has been parallelised.
subroutine resid(idir,nl,flux,res)

implicit none

include 'common.block'
include 'cosa.inc'

integer*4 idir,nl,iblock,imax,jmax,iflux,ires
real (kind=8) res(*),flux(*)

do iblock = 1,mynblocks
  imax = i_imax (iblock,nl)
  jmax = j_jmax (iblock,nl)
  iflux = 1 + off_p3 (iblock,nl) * npde * dim5
  ires = 1 + off_0 (iblock,nl) * npde * dim5
  call bresid(flux(iflux),res(ires),idir,imax,jmax,npde,nharms)
end do

return
end

subroutine bresid(flux,res,idir,imax,jmax,npde,nharms)

implicit none

include 'common.block'

integer*4 imax,jmax,npde,nharms
integer*4 i,j,idir,ipde,n
real (kind=8)
&     res(-1:imax+1,-1:jmax+1,npde,0:2*nharms),
&     flux(imax,jmax,npde,0:2*nharms)

if (idir.eq.1) then
  !$OMP PARALLEL DO
  !$OMP& PRIVATE(n,ipde,j,i)
  !$OMP& SHARED(nharms,npde,cosa2d,cosa3d,imax,jmax,res,flux)
  #ifdef OPENMPNHARMS
  !$OMP!
  !$OMP&!
  !SPARALLEL DO
  !$OMP& PRIVATE(n,ipde,j,i)
  !$OMP& SHARED(nharms,npde,cosa2d,cosa3d,imax,jmax,res,flux)
  #ifdef OPENMPNHARMS
  !$OMP!
  !$OMP&!
  !END PARALLEL DO
  #endif
  do n = 0,2*nharms
    do ipde = 1,npde
      if ((cosa2d.and.(ipde.ne.4)).or.cosa3d) then
        do j = 1,jmax+1
          do i = 1,imax-1
            res(i,j,ipde,n) = res(i,j,ipde,n) +
            & flux(i+1,j,ipde,n)-flux(i,j,ipde,n)
          end do
        end do
      end if
    end do
  end do
  !$OMP END PARALLEL DO
  #endif
  else if (idir.eq.2) then
  !$OMP PARALLEL DO
  !$OMP& PRIVATE(n,ipde,j,i)
  !$OMP& SHARED(res,flux,cosa3d,npde,nharms,cosa2d,imax,jmax)
  #ifdef OPENMPNHARMS
  !$OMP!
  !$OMP&!
  !SPARALLEL DO
  !$OMP& PRIVATE(n,ipde,j,i)
  !$OMP& SHARED(res,flux,cosa3d,npde,nharms,cosa2d,imax,jmax)
  #ifdef OPENMPNHARMS
  !$OMP!
  !$OMP&!
  !END PARALLEL DO
  #endif
  do n = 0,2*nharms
    do ipde = 1,npde
      if ((cosa2d.and.(ipde.ne.4)).or.cosa3d) then
        do j = 1,jmax+1
          do i = 1,imax-1
            res(i,j,ipde,n) = res(i,j,ipde,n) +
            & flux(i+1,j,ipde,n)-flux(i,j,ipde,n)
          end do
        end do
      end if
    end do
  end do
  !$OMP END PARALLEL DO
  #endif
  else if (idir.eq.3) then
  !$OMP PARALLEL DO
  !$OMP& PRIVATE(n,ipde,j,i)
  !$OMP& SHARED(res,flux,cosa2d,npde,nharms,cosa3d,imax,jmax)
  #ifdef OPENMPNHARMS
  !$OMP!
  !$OMP&!
  !SPARALLEL DO
  !$OMP& PRIVATE(n,ipde,j,i)
  !$OMP& SHARED(res,flux,cosa2d,npde,nharms,cosa3d,imax,jmax)
  #ifdef OPENMPNHARMS
  !$OMP!
  !$OMP&!
  !END PARALLEL DO
  #endif
  do n = 0,2*nharms
    do ipde = 1,npde
      if ((cosa3d.and.(ipde.ne.4)).or.cosa2d) then
        do j = 1,jmax+1
          do i = 1,imax-1
            res(i,j,ipde,n) = res(i,j,ipde,n) +
            & flux(i+1,j,ipde,n)-flux(i,j,ipde,n)
          end do
        end do
      end if
    end do
  end do
  !$OMP END PARALLEL DO
  #endif
  else if (idir.eq.4) then
  !$OMP PARALLEL DO
  !$OMP& PRIVATE(n,ipde,j,i)
  !$OMP& SHARED(res,flux,cosa2d,npde,nharms,cosa3d,imax,jmax)
  #ifdef OPENMPNHARMS
  !$OMP!
  !$OMP&!
  !SPARALLEL DO
  !$OMP& PRIVATE(n,ipde,j,i)
  !$OMP& SHARED(res,flux,cosa2d,npde,nharms,cosa3d,imax,jmax)
  #ifdef OPENMPNHARMS
  !$OMP!
  !$OMP&!
  !END PARALLEL DO
  #endif
  do n = 0,2*nharms
    do ipde = 1,npde
      if ((cosa3d.and.(ipde.ne.4)).or.cosa2d) then
        do j = 1,jmax+1
          do i = 1,imax-1
            res(i,j,ipde,n) = res(i,j,ipde,n) +
            & flux(i+1,j,ipde,n)-flux(i,j,ipde,n)
          end do
        end do
      end if
    end do
  end do
  !$OMP END PARALLEL DO
  #endif
  else return
  end

Below is an optimised version of the above routine where the OpenMP parallel region has been hoisted to the routine above to reduce the OpenMP overheads.

```fortran
implicit none
include 'common.block'
include 'cosa.inc'
integer*4 idir,nl,iblock,imax,jmax,iflux,ires
real (kind=8) res(*),flux(*)

#ifdef OPENMPNHARMS
!$OMP PARALLEL DEFAULT(NONE)
!$OMP& SHARED(mynblocks, i_imax, j_jmax, off_p3, off_0, nharms, nl, npde, dim5)
!$OMP& SHARED(flux, res, idir)
!$OMP& PRIVATE(imax, jmax, ires, iflux, iblock)
#endif
iblock = 1,mynblocks
imax   = i_imax     (iblock,nl)
jmax   = j_jmax     (iblock,nl)
ires   = 1 + off_p3 (iblock,nl) * npde * dim5
iflux  = 1 + off_0  (iblock,nl) * npde * dim5
end do

#ifdef OPENMPNHARMS
$OMP END PARALLEL
#endif
return
end

subroutine bresid(flux,res,idir,imax,jmax,npde,nharms)
implicit none
include 'common.block'
integer*4 imax,jmax,npde,nharms
integer*4 i,j,idir,ipde,n
real (kind=8)
&     res(-1:imax+1,-1:jmax+1,npde,0:2*nharms),
&     flux(imax,jmax,npde,0:2*nharms)
if (idir.eq.1) then
#ifdef OPENMPNHARMS
$OMP DO
$OMP& PRIVATE(n,ipde,j,i)
#endif
dn = 0,2*nharms
do ipde = 1,npde
if (((cosa2d.and.(ipde.ne.4)).or.cosa3d) then
do j = 1,jmax-1
do i = 1,imax-1
res(i,j,ipde,n) = res(i,j,ipde,n) +
&     flux(i+1,j,ipde,n)-flux(i,j,ipde,n)
end do
end do
else if (idir.eq.2) then
#ifdef OPENMPNHARMS
$OMP DO
$OMP& PRIVATE(n,ipde,j,i)
#endif
dn = 0,2*nharms
do ipde = 1,npde
if (((cosa2d.and.(ipde.ne.4)).or.cosa3d) then
do j = 1,jmax-1
do i = 1,imax-1
res(i,j,ipde,n) = res(i,j,ipde,n) +
&     flux(i+1,j,ipde,n)-flux(i,j,ipde,n)
end do
end do
#endif
#endif
endif

end subroutine bresid
```
Appendix B
The configuration data for each of the test cases is included below.

Test Case 1

*** input file for 2D Euler/NS COSA solver ***
model id flow-type debug
laminar 1 external n
gamma reyno pranl machfs alpha
1.4d0 1100 0.71d0 1.d-01 0.d0
flow-mode solver rk option nharms freq.
unsteady hb rke x 0.014
move dh0x dh0y xh yh dthet0
phipp plupit 0.d0 1.d0 0.33333333 0.d0 76.33d0
nst d0
irest srest cfl lmax iupdt toler
0 1500 0.5 250 1 1.d-20
rkap irs-typ cfl psi
-1. cirs_v1 1.0 0.25
ilim epslim cntrpy etpfxtyp entfxctf
0 5.d-04 0.d0 0 0.95d0
nivel nl_crs nl_fmg nstart npre npost
nlevel 3 2 1 5 5 5
10
prol. type bilinear
flow-speed
nolomach
tref
288.2
functional
cl_cd_cm_1 0.33333333 0.0
1
20 475 476 479 480 483 484 487 488 491 492 495 496 499
500 503 504 507 508 511 512
512

Test Case 2
Coarse input file:

*** input file for 2D Euler/NS COSA solver ***
model id flow-type debug
laminar 1 external n
**Fine input file:**

```plaintext
*** input file for 2D Euler/NS COSA solver ***
model id flow-type debug
laminar 1 external n
gamma reyno pranl machfs alpha
1.4d0 1000 0.71d0 0.2244 9.138
flow-mode solver rk option nharms freq.
unsteady hb rkex 8
0.002707149187620
move dh0x dh0y
plunge 1.2091 0.d0
irest srest cfl rmpf lmax iupdt
toler 0 4000 1.5 1.0 10000 1
1.d-13
rkap irs-typ resper cfli psi
-1. cirs_v1 0 3.0d0 0.25
ilim epslim cntrpy etpfxtyp entfxctf
0 5.d-04 0.d0 0 0.95d0
nlevel nl_crs nl_fmg nstart npre npost
crs
3 4 1 2 2
8
flow-speed cff-typ visc-prec prec-typ lx ly
lomach 2 viscous mixed 28.0 28.0
epsp(1) 9.d-1 9.d-1
2.d-1
288.2
functional
clmom 0.25d0 0.d0
1 ! nbody
12 3 4 5 6 7 8 9 10 11 12 13 14 ! n_bodblo,
i_bdbl(i_bodblo)
32
```
Test Case 3

*** input file for 2D Euler/NS COSA solver ***

model id flow-type debug
  laminar 1 external n
  gamma reyno pranl machfs alpha
  1.4d0 1000 0.71d0 0.2244 9.138
  flow-mode solver rk option nharms freq.
  unsteady hb rkex 8
  0.002707149187620
  move dh0x dh0y
  plunge 1.2091 0.d0
  irest srest cf1 lmax iupdt toler
  0 200 1.5 2 1 1.d-12
  rkap irs-typ cfli psi
  -1. cirs_v1 3.0d0 0.25
  ilim epslim cntrpy etpfxtyp entfxctf
  0 5.d-04 0.d0 0 0.95d0
nlevel nl_crs nl_fmg nstart npre npost
  ncrs
  3 4 1 2 2 2
  8
  flow-speed cff-typ visc-prec prec-typ lx ly
  lomach 2 viscous mixed 28.0 28.0
  epsp(1) epsp(2) epsp(3)
  2.d-1 9.d-1 9.d-1
tref 288.2
  functional
c1_cd_cm_0 0.25d0 0.d0
  1